

# Incompressible Quantum Glass state of Bosons

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in Collaboration with

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# outline

Introduction

QMC simulations of incompressible QG in 2D

Theory: a percolation scenario

Compressible and incompressible QG

Summary and discussions

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## Introduction

QMC simulations of incompressible QG in 2D

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Compressible and incompressible QG

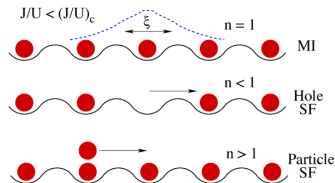
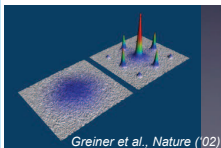
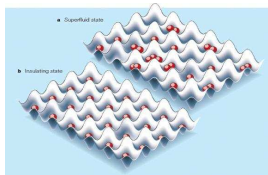
Summary and discussions

# Interacting lattice bosons: clean system

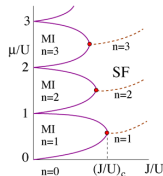
$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \frac{U}{2} \sum_{i=1}^N n_i(n_i - 1) - \mu \sum_{i=1}^N n_i,$$

Two ways from MI to SF

Two ground states SF and MI



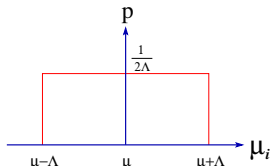
- ▶ MI: integer filling, insulating, gapped
- ▶ SF: any filling fraction, gapless



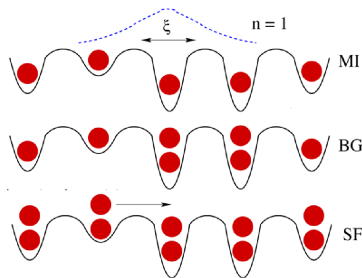
# Interacting lattice bosons: site-disorder present

**Fascinating interplay between disorder, interactions and SF**

$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \frac{U}{2} \sum_{i=1}^N n_i(n_i - 1) - \sum_{i=1}^N \mu_i n_i,$$

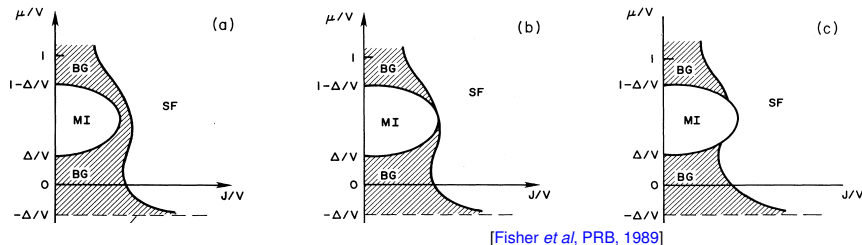


- ▶ SF: any filling fraction, gapless
- ▶ MI: integer filling, insulating, gapped
- ▶ **Quantum Glass**: insulating but **Gapless**;  
**believed**: always compressible (BG)  
in 2D



# Previous studies of the site-disordered BH model

$T = 0$  phase diagram: QG state always intervenes SF and MI ?



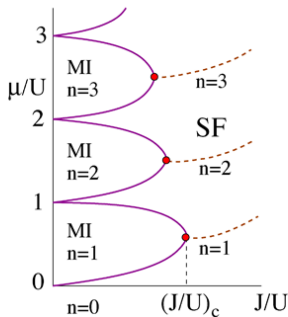
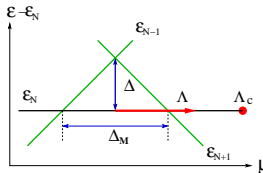
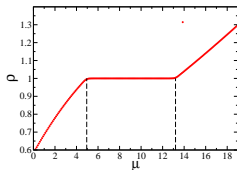
Söyler '11; Prokof'ev, '04; Pollet, '09; Herbut, '97; '98; Weichman, '96, '08; Svistunov, '96;

Singh; '92; Pazmandi; '98; Pai, '96;

Scalettar, '91; Krauth, '91; Kisker; '97; Sen, '01; Lee, '01; Wu, '08; Bissbort, '09

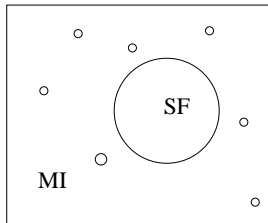
# Recent progress

QG is a Griffiths phase in which **rare large regions** of phase A inside phase B lead to singularities.



- ▶  $\Lambda > \Delta_M/2$ , arbitrarily large regions of SF inside the MI.
- ▶  $\Lambda < \Lambda_c$ , SF puddles NOT percolating, insulating.
- ▶ Fundamentally different from the MI:  
gapless due to arbitrarily large SF region

Pollet *et al*, PRL, 2009

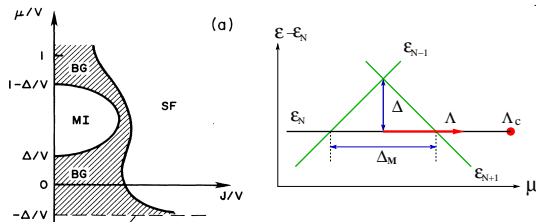




# Recent progress

## Phase diagram

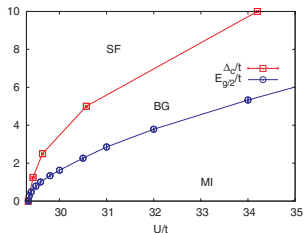
$\mu/U$  vs.  $t/U$  for given  $\Lambda$



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$\Lambda$  vs.  $U/t$  for  $\rho = 1$ .



MI-QG boundary can **NOT**  
be extracted basing on  
QMC

A theoretical curve

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# Our work: the temperature behavior of $\kappa$

- We find  $\kappa$  in certain region of QG:

$$\kappa \sim \exp(-b/T^\alpha), \quad \alpha < 1$$

$\kappa \rightarrow 0$  when  $T \rightarrow 0 \Rightarrow$  **Mott Glass**

MI: the above form with  $\alpha = 1$  and  $b = \Delta$

**In contrast to the commonly accepted theory of site-disordered bosons**

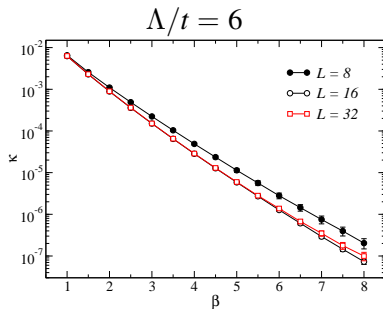
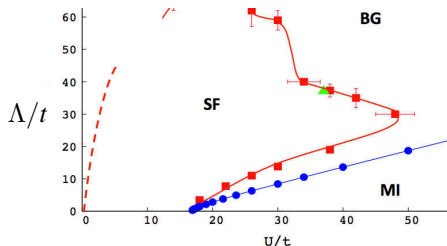
**Gapless does not necessarily imply a compressible state**

You can add or remove a particle with infinitesimal cost, but the total amount of the cost can be zero comparing to system size  $N$

- such **MG** only found in 1D system with explicit P-H symmetry, e.g., hopping disorder Altman *et al*, PRL, 2004.

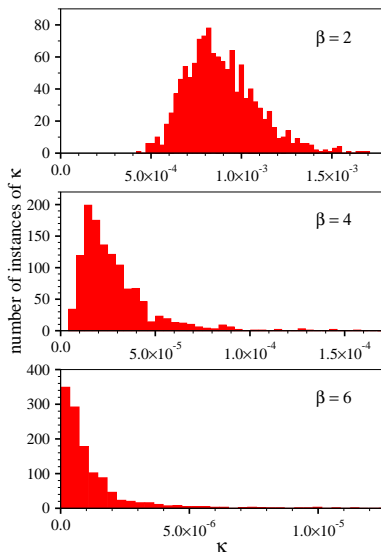
# QMC simulations

- ▶ method: SSE QMC with directed-loop updates
- ▶ Adjust  $\mu$  to ensure  $\rho = \langle n \rangle / N = 1$
- ▶ Fix  $U/t = 22$ , **2D**: QG for  $4.2 < \Lambda < 7.8$
- ▶ Average over up to thousands of realizations
- ▶ For  $\beta = t/T$  up to 8, eliminate finite-size effects by using  $L$  up to 32.

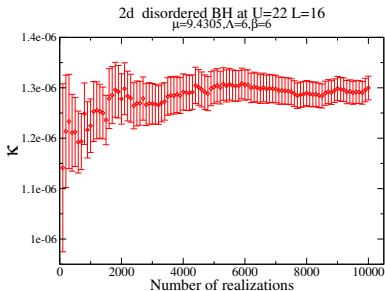


Söyler *et al*, 2011

# QMC simulations: Low- $T$ $\kappa$ distribution

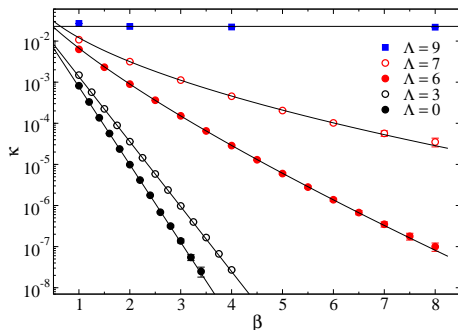


Difficulty:  $\kappa$  distribution develops a stretched tail  
But it is not sufficiently fat to cause serious problems



$L = 16$ , 1200 realizations

# QMC simulations : $\kappa - T$ behavior



- MI points ( $\Lambda = 0$  and 3): agrees with the conventional  $\alpha = 1$  exponential

$$\kappa \propto e^{-\Delta/T}$$

- $\kappa$  in **QG (MG)** follows the exponential form

$$\kappa \sim \exp(-b/T^\alpha)$$

$$\alpha \approx 0.77 \text{ for } \Lambda = 6$$

$$\alpha \approx 0.53 \text{ for } \Lambda = 7$$

- SF point ( $\Lambda = 9$ ):  $\kappa(T)$  converges rapidly to a non-zero value

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# “QG should be a compressible BG” may fail, why?

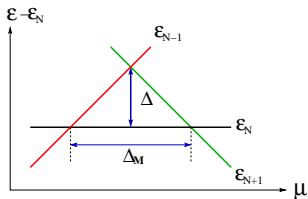
The compressibility is normalized by the system volume!

$$\kappa = \frac{\partial \rho}{\partial \mu} = \frac{1}{TN} \frac{\sum_n \int d\epsilon e^{-\epsilon/T} \rho_n(\epsilon) (n - \langle n \rangle)^2}{\sum_n \int d\epsilon e^{-\epsilon/T} \rho_n(\epsilon)}$$

$\rho_n(\epsilon)$ : density of states with  $n$  bosons.



## MI state of a clean system:



Gap  $\Delta = \frac{\Delta_M}{2}$  to adding (removing) a boson.

The added particle disperses as a free boson (large  $U$ ),

$$\epsilon_q = \Delta + 4t - 2t[\cos(q_x) + \cos(q_y)]$$

At low  $T \rightarrow$  flat density of states in a band of width  $W$

$$\rho_{N\pm 1}(\epsilon) = \begin{cases} N/W & , \epsilon \in [\Delta, \Delta + W] \\ 0 & , \text{otherwise} \end{cases}$$

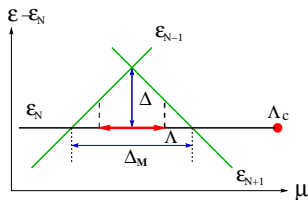
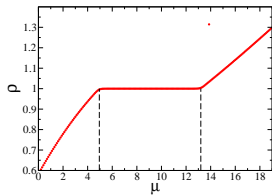
Only consider the  $n = N$  sector in the denominator,

$$\kappa = \frac{2}{TN} \int_{\Delta}^{\Delta+W} d\epsilon e^{-\epsilon/T} \frac{N}{W} (N \pm 1 - N)^2 = \frac{2}{W} e^{-\Delta/T} (1 - e^{-W/T}) \approx \frac{2}{W} e^{-\Delta/T}$$

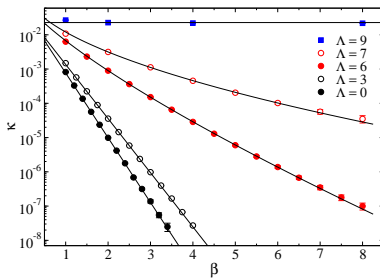
# MI, Comparing with QMC simulations

$$U/t = 22, \rho = 1, \Delta_M \approx 8.34$$

- Clean system: The pure exponential form agrees with QMC. We obtain  $\Delta \approx 4.2$



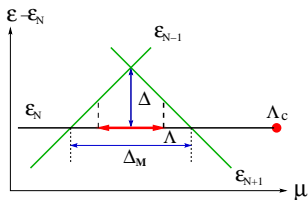
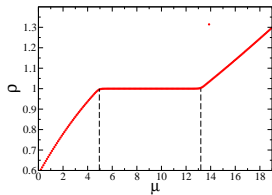
$$\kappa_{MI} \approx \frac{2}{W} e^{-\Delta/T}$$



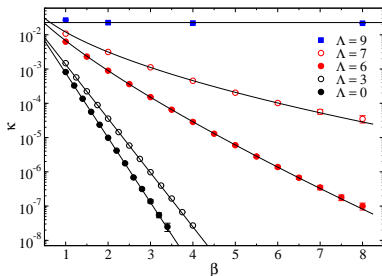
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- Weak disordered system ( $\Lambda > 0$ ),  $\Delta \rightarrow$  **averaged gap**  
The pure exponential form agrees with our QMC for  $\Lambda = 3$ .

# QG state

**Gapless**: GS no longer dominates, can be ignored in the partition function, however

- Contribution to  $\kappa$  from the  $N \pm 1$  sectors:

$$\kappa_1 \approx \frac{2}{TN} \frac{\int d\epsilon e^{-\epsilon/T} \rho_{n\pm 1}(\epsilon) (\pm 1)^2}{\sum_{n=N\pm 1} \int d\epsilon e^{-\epsilon/T} \rho_n(\epsilon)} \propto \frac{1}{TN} \rightarrow 0$$

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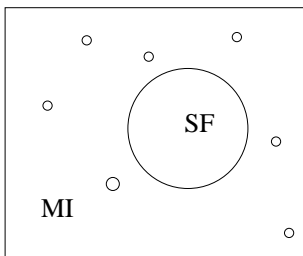
- Contribution from other sectors?

To get  $\kappa$  size independent, need enough low-energy states in sectors with  $|n - N| \sim \sqrt{N}$

This is the case for SF state, but may not be the case for a QG due to **finite size gaps of SF domains in 2D**

# Percolation scenario

Consider an ensemble of SF domains **below the percolation threshold**



thus

$$\kappa \propto \exp(-bT^{-c/a}) = \exp(-bT^{-\alpha})$$

- ▶ Finite-size ( $m$ ) gap  $\propto \frac{1}{m^a}$ ,  $a$  unknown exponent.
- ▶ Given  $T = \frac{1}{m^a}$ , all domains of sizes  $s < m$  gapped, **NO contribution to  $\kappa$**
- ▶ Only domains of sizes  $s > m$  contribute to  $\kappa$   
Prob. of a site belong to an SF domain with  $s > m$  is  $\propto \exp(-bm^c)$ ,  $c$  unknown exponent.

# Cross-over behavior

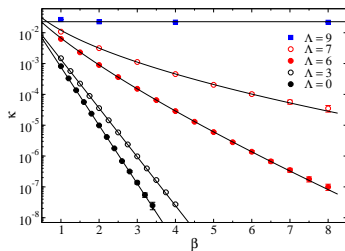
Always a contribution from the MI background,

$$\kappa_{MI} \propto \exp(-\Delta/T), \quad \Delta: \text{average gap}$$

total

$$\kappa \propto \kappa_{MI} + \exp(-bT^{-\alpha})$$

►  $\kappa_{MI}$  negligible at low  $T$  when  $\alpha < 1$



# Cross-over behavior

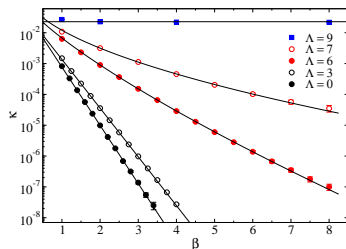
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- ▶ Approaching the MG–MI transition from MG side,  $b$  diverges.  
 $\kappa$  crosses over to  $\kappa_{MI}$ , i.e.,  $\alpha = 1$





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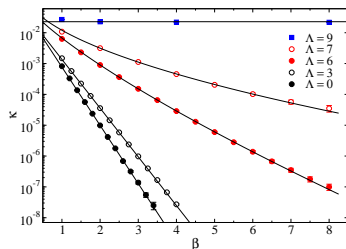
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- ▶ Approaching the MG–MI transition from MG side,  $b$  diverges.  
 $\kappa$  crosses over to  $\kappa_{MI}$ , i.e.,  $\alpha = 1$
- ▶ Approaching the MG–SF transition from MG,  $\alpha \rightarrow 0$ .  
 $\kappa$  evolves smoothly between the MG form and the constant  $\kappa$  at the transition



- ▶ Jump happens at  $T = 0$

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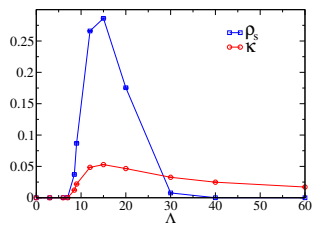
**Compressible and incompressible QG**

Summary and discussions

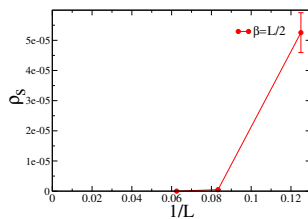
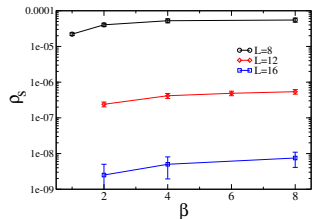
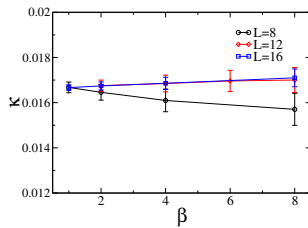
# Compressible QG state (2D)

For very large  $\Lambda$ , the system enter QG state again, but compressible

$$\beta = 8, L = 16$$

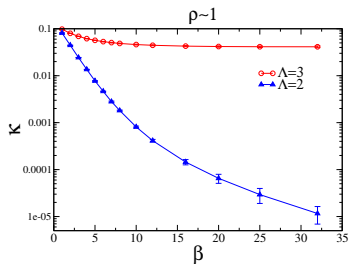
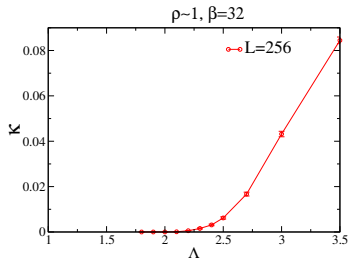
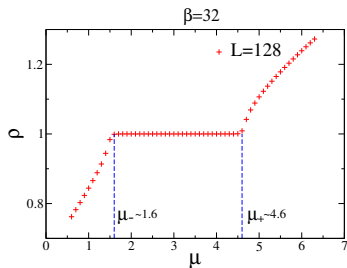


At  $\Lambda = 60$



# Incompressible and Compressible QG state in 1D

Take  $U/t = 8$  as an example,  $\Delta_M/2 \approx 1.5$



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- ▶ Based on plausible arguments and unbiased QMC results: there is an incompressible QG phase at  $T = 0$  for commensurate filling and moderate disorder strength.

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- ▶ Mechanism: finite-size gap of finite SF domains surrounded by an MI. The gap decreases with increasing size of the SF domain, but as long as the SF do NOT percolate, lead to incompressibility in  $T = 0$ .
- ▶ The factor  $1/N$  in  $\kappa$ : number of particles can be added, but may NOT enough to offer a Nonzero  $\kappa$  at  $T \rightarrow 0$ .



# Conclusions and discussion

- ▶ For extremely large disorder strength, the percolation scenario doesn't apply.

- ▶ Our QMC for  $\rho = 1$ .

Commensurate fillings might be special in 2D

There is an emergent particle-hole symmetry at integer  $\rho$

- ▶ The percolation picture may does not apply at noninteger fillings.

A compressible BG might exist at noninteger filling.

Weak-disorder RG: 2D integer filling have a different fixed point.

Krüger, PRB 2011

Others: no distinction

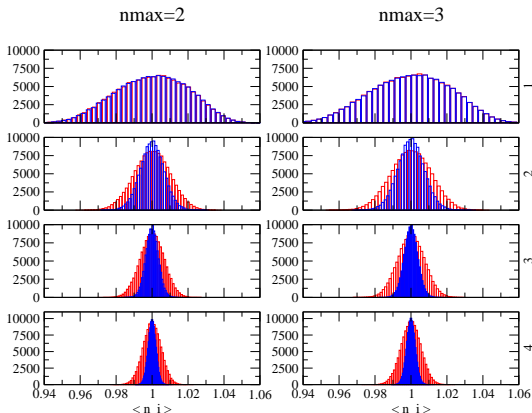
Fisher PRB 89, Pollet PRL 09

Thank You!

The **emergent** particle-hole symmetry at integer  $\rho$  and makes these systems special.

Blue: histograms of  $\langle n_i \rangle$  in  $l \times l$  box;

Red: histograms obtained by randomly averaging  $l \times l$   $\langle n_i \rangle$   
( $\beta = 8, 500$  realizations.)



# Even lower temperature

