Case with more significant corrections the possibility of competing scaling corrections at quantum critical points. induces the next correction, which has well as well and **Case with more significant corrections**

- common at quantum critical points One of the best understood quantum phase transi-- common at quantum critical points $\begin{array}{ccc} \hline \text{...} & \hline \end{array}$
- S=1/2 Heisenberg model with \sim 1/0 Heiecakese model with Were not always a shown and the same (CDM)
- columnar dimers (CDM) \sim Columna uniters. (ODM)
- and there is an and the set of the stage dimerrial (2DNA) models with inter-- staggered dimers (SDM) staggered differs (SDM)

The SDM has been controversial \longrightarrow

- O(3) or new universality class and the spinal review of the spinal review of the spinal review of the value o
- strange scaling behaviors example of this transition \mathbf{r}

2.28

PHYSICAL REVIEW LETTERS 121, 117202 (2018) (2018)

that of the formally leading conventional conventional correction with exponent α , α , α

Anomalous Ouantum-Critical Scaling $\frac{1,2,3}{\sqrt{1-\frac{1}{2}}}$ Dhillin Wainbarg $\frac{3}{2}$ Hui Sheeld driven transition of the Members, that structure $\ddot{}$ · Si $\ddot{}$, of strength (prefactor) $\ddot{}$, or strength (prefactor) $\ddot{}$ rrecuons in 1 wo-Dhhensional Anuier $\frac{1}{2}$ tum phase transition with dynamic exponent $\frac{1}{2}$ Anomalous Quantum-Critical Scaling Corrections in Two-Dimensional Antiferromagnets Nysen Ma,^{1,2,3} Phillip Weinberg,³ Hui Shao,^{4,3} Wenan Guo,^{3,4} Dao-Xin Yao,^{1,*} and Anders W. Sandvik^{3,2,†} $\frac{1}{2010}$ Anomalous Quantum-Critical Scaling Corrections in Two-Dimensional Antiferromagnets Nvsen Ma,^{1,2,3} Phillip Weinberg,³ Hui Shao,^{4,3} Wenan Guo,^{5,4} Dao-Xin Yao,^{1,*} and Anders W. Sandvik^{3,2,†} $\ddot{}$

(columnar) dimerized model, where cubic interactions are $(2\pi)^{-1}$ present. We conclude that there is a new conclude that there is a

irrelevant field in the staggered model, but, at variance with previous claims, it is not the leading irrelevant $\overline{}$ and the new exponent is $\overline{}$ and the prefactor of the correction L $\overline{}$

irrelevant field in the staggered model, but, at variance with previous claims, it is not the leading irrelevant

Analyze critical behavior with two scaling **EXECUM Analyze critical behavior with two scaling Corrections taken into account** 2.40 $\left[\begin{array}{cc} \Delta & L=16 \\ 2.40 & \Delta & L=32 \end{array}\right]$ FIONS LAKEN INTO ACCOUNT 170 oritical behavior with two coaling wo scanny ections taken into account the condense of $2.40\frac{1}{\alpha}$ $\frac{L=16}{L=32}$ and $\frac{1}{\alpha}$ $\frac{1}{\alpha}$ wo scaling \overline{a} \mathbf{z}

$$
O(g,L)=f[(g-g_c)L^{1/\nu},\lambda_1 L^{-\omega_1},\lambda_2 L^{-\omega_2},\cdots]\qquad \overset{\text{2.36}}{\circ} \left[\begin{array}{cc} \text{v} & L=128 \\ \text{o} & L=256 \end{array}\right])
$$

quantization axis, the L-normalized spin stiffness con-

QMC and fitting procedures.—We use the standard

 $s_{\rm eff}$ and $\alpha_{\rm eff}$

Touler expend speluze ereceipe m of the spins of the spins of spins α . for different difficits ionicss quartu Taylor expand, analyze crossing points for different dimensionless quantities **state of the CDM.** 2.28 ful dimensionless quantities to study in QMC calculations and, analyze crossing points $\begin{array}{cc} \mathbb{R} & \mathbb{R} \end{array}$

have been tested this way [11–16]. Compare CDM and SDM behaviors $\overline{}$ Compare CDM and SDM behaviors $2.24\frac{1}{2.516}+\frac{1}{2.518}+\frac{1}{2.520}+\frac{1}{2.518}$ SDM and SDM behaviors and the role of the $\frac{2.24\frac{1}{2.516} + \frac{1}{2.518} + \frac{1}{2.520}}{2.520}$ $\frac{1}{2.522}$

tions surrounding the 2D AFM–paramagnetic transition.

 A long-standing unresolved is differences observed is differences observed is differences observed is differences observed in \mathcal{A}

tem size L can appear in two pairs, (L, 2L) as well as

mized as pect ratios (21) have convincing as perfect ratios μ

that there is no new universality class, the reasons for the reasons for the reasons for the reasons for the r

the neighborhood of gc. The curves are polynomial fits giving are polynomial fits giving σ

Leading-order cross-point sl Leading-order cross-point shifts 2.52^{Fe} $1 \cdot e \cdot \text{div} \cdot \text{div$ **Leading-order cross-point shift**

one can derive simple simple simple simple expressions for the crossing values α

$$
g^*(L) = g_c + aL^{-\omega_1 - 1/\nu},
$$

$$
O^*(L) = O_c + bL^{-\omega_1},
$$

actually not unexpected within the scenario of irrelevant - WORKS TOR CLIWI, $\omega_1 \approx 0.78$ r_{e} $\frac{1}{2}$ voiks lui CDIVI, W1 \approx 0.1 - Works for CDM, $ω₁≈0.78$

 ϵ gauge gauge gauge gauge ϵ . ϵ 1.31945(1), 1.31, 1.30(1), 1.30(1), 1.30(1), 1.30(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1), 1.3(1)

ing corrections are used to first order, with α and α and α and α and α and α and α

fitted critical point is graded corresponding in \mathcal{L}^2

(though not much outside one error bar of the difference).

The key result here is clearly that ω² comes out larger

- **FROM BE PRESENT STILL BE PRESENT TWO COMPECTIONS NEEDED TO THE FR** ω ₁≈0.78, ω ₂≈1.25 - Two corrections needed for SDM $\omega_1 \approx 0.78$, $\omega_2 \approx 1.25$
- Fits within theory where the SDM field theory needs a new term show (Fritz at least \overline{r} \mathbf{r} - \mathbf{r} (Fritz et al, PRB 2012) $r = r \cdot \frac{1}{2}$ shown with $r = r \cdot \frac{1}{2}$ shown with $r = r \cdot \frac{1}{2}$ **Produce various interest values** $\frac{1}{2}$

FIG. 5. Size dependence of the exponent of the exponent of the exponent in as defined in as defined in a
The exponent of the exponent o

first correction term in Eq. (7), with α and β fixed. In Eq. (7), with α is defined. In Eq. (7), with α

ery of the anomalous behaviors for the SDM [17], all the

ω² = 1.29(5) is the result of the fit.

Cruer parameter at the critical point \mathbb{R}^m can then define a size-dependent as size-dependent as size-dependent as \mathbb{R}^m ing determined a precise estimate of gc, we study the Order parameter at the critical point

$$
\left\{\n\begin{array}{ll}\n\langle m^2 \rangle_c \propto L^{-(1+\eta)} (1+b_1 L^{-\omega_1} + b_2 L^{-\omega_2} + \ldots)\n\end{array}\n\right.
$$

$$
\eta^*(L) = \ln[\langle m^2(L) \rangle_c / \langle m^2(2L) \rangle_c] / \ln(2) - 1
$$

To test this form and extract was the known value of the known value of this contract was the known value.
The known value of the known value

 $\mathcal{L}_{\mathcal{A}}$. Later we will also the contribution of $\mathcal{L}_{\mathcal{A}}$. Later we will argue that $\mathcal{L}_{\mathcal{A}}$

$$
\eta^*(L) = \eta + c_1 L^{-\omega_1} + c_2 L^{-\omega_2} + \ldots
$$

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Προσεργασία

Field-theory aspects of the anomalous scaling [Fritz et al., PRB 2011] <u>Field-</u>
The S of the anomalous scaling **Series at all PRR 2011** Of the anomalous scaling [Fritz et al., PRB 2011] **described by** *S*
Pield-theory aspects of the anomalo <u>Field-theory aspec</u> **Field-theory aspects of the anomalous s Field-theory aspects of the anomalous s**
The SDM leads to seek in terms in the field (b) Eight theory consets of the gramoleus see <u>Figue and the discussions in the discussion terms in the terms are the the terms and the terms are th</u> **B. Monte Carlo analysis in** *D* = **3** $\lim_{n\to\infty}$ isotropic, the action in *^D* ⁼ *^d* ⁺ 1 dimensions reads⁶ heory action

The SDM leads to a cubic term in the field-theory action The SDM leads to a cubic term in the field the Wolff requester algorithm, and note The SDM leads to a cubic term in the field-theory action *d x* action

We start by determining the scaling dimension of the cubic

 $2\Delta_{\cal{O}}$

 $\mathcal{C}(t) = \mathcal{C}(t)\mathcal{C}(0)/\propto \frac{1}{|\vec{r}|^{2\Delta_0}}$

$$
S = \frac{1}{2} \int d^D r \left[m_0 \varphi_\alpha^2 + (\vec{\nabla}\varphi_\alpha)^2 \right] + \frac{u_0}{4!} \int d^D r \left(\varphi_\alpha^2 \right)^2 + i \gamma_0 \int d^D r \, \vec{\varphi} \cdot (\partial_x \vec{\varphi} \times \partial_y \vec{\varphi})
$$

Is the cubic term relevant or irrelevant at the O(3) critical point? $\mathcal{O}(\vec{r}) = \vec{\varphi}(\vec{r}) \cdot (\partial_x \vec{\varphi}(\vec{r}) \times \partial_y \vec{\varphi}(\vec{r}))$ Is the cubic term relevant or irrelevant at the $C(\vec{r}) = \langle \mathcal{O}(\vec{r})\mathcal{O}(0)\rangle \propto$ 1 $g(\vec{r})$ $(\Omega(\vec{r})/\Omega(0))$ or $\frac{1}{\Omega(\vec{r})}$ $(\Omega(\vec{r}) \rightarrow \vec{r})$ $C(\vec{r}) = \langle \mathcal{O}(\vec{r})\mathcal{O}(0)\rangle \propto \frac{1}{|\vec{r}|^{2\Delta_{\mathcal{O}}}}$ $\mathcal{O}(\vec{r}) = \vec{\varphi}(\vec{r})$ 0*.*693 035(37).38 + *i*γ⁰ !
! *a D a D D <i>d d d d d d dimensions* we obtain the O(3) critical point? boils down to the simulation of a *classical* problem in *D* = \cdot $(\partial_x \vec{\varphi}(\vec{r}) \times \partial_y \vec{\varphi}(\vec{r}))$

MC of the classical Heisenberg model to extract scaling dimension perisong-moder to extraot obaling almo. $|\vec{r}|$ MC of the classical Heisenberg model to ex- \mathcal{C} FRICAL REVIEW B REVIEW B 83, 1744 1999, Western B 83, 174416 (2011) *BDC 174416 (2011)* [ϕ⃗]G = (*D* − 2)*/*2*,* \overline{a} MC of the classical Heisenberg model to extract scaling dimension

$$
H = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j
$$

$$
\mathcal{O}_{i, \text{lattice}} = \vec{S}_i \cdot (\vec{S}_{i+e_x} \times \vec{S}_{i+e_y})
$$

$$
\vec{S}_i \cdot (\vec{S}_{i+e_x} \times \vec{S}_{i+e_y})
$$

$$
\begin{array}{c}\n\text{or} \\
\vec{S}_i\n\end{array}
$$

(*d* + *z*) dimensions, with *z* = 1, as the O(3) critical field theory

with ferromagnetic interactions between ϵ interactions between nearest nearest nearest nearest nearest near

They found $\Delta_{\rm o}\sim 3.2\,...\,3.5$ **They found** $\Delta_0 \sim 3.2 \dots 3.5$
O O b_{\widehat{Q} 10} They found $\Delta_0 \sim 3.2 ... 3.5$ $\frac{10}{100}$ is and $\frac{10}{100}$ on $\frac{10}{100}$ They found $\Delta_0 \sim 3.2 \ldots 3.5$

- **receive corresponds to** $\omega \sim 0.2 0.5$ $\text{corresponds to } \omega \sim 0.2 - 0.5$ \sim corresponds to $\omega \sim 0.2 - 0.5$
- \overrightarrow{a} would be leading correction **Before would be leading correction** σ ⁻⁷ - would be leading correction
- $\frac{d}{dt}$ Inconsistent with SI at critical is in the model is i
Monte Carlo simulation and provides high-accuracy critical is in the model in the model is in the model in th $\frac{3}{2}$ as the O(3) $\frac{3}{2}$ ~ 1.3 universality contribution class. make a brief detailed a brief. A single of the single process the single state of the single state of the scaling of $\frac{1}{2}$ at the Wilson-Fisher with spin consistent with $\frac{1}{2}$ Inconsistent with SDM QMC
- amove <u>α</u> − 1.3 $-\omega \sim 1.3$

where $\mathcal{L}_{\mathcal{S}}$ is $\mathcal{L}_{\mathcal{S}}$ in $\mathcal{L}_{\mathcal{S}}$ in $\mathcal{L}_{\mathcal{S}}$ was used. Although $\mathcal{L}_{\mathcal{S}}$

isotropic, the action in *^D* ⁼ *^d* ⁺ 1 dimensions reads⁶

 $-$ second correction $=$ second correction

H = −*J*

temperature. The dashed line corresponds to a decay proportional to a decay proportional to a decay proportional to \mathcal{A}

and hence realizes the Wilson-Fisher fixed point in *D* = 3, but

gives us access to correlation functions in a nonperturbative α

≈ −0*.*556 25*,* (27) athod, There is an approtor with \sim 1.1 salou. There is an operator with $\omega \sim 1.4$ Conformal bootstrap method: There is an operator with $\omega \sim 1.4$ Uomomial bootstrap method: There is an operator of the operator **O** α improved classical MC would be useful functions of *L/*2 for different system sizes *L* from Monte Carlo Ω and experience that the correlation of the correlation is the correlation of the cor \mathbf{v} at criticality. The model is in the m = second correction
Conformal bootstrap method: There is an operator with $ω \sim 1.4$

^j (33)

and hence realizes the Wilson-Fisher fixed point in D $=$ 3, D - improved classical ivic would be useful \mathcal{F}_1 , \mathcal{F}_2 , \mathcal{F}_3 , \mathcal{F}_4 , \mathcal{F}_5 , \mathcal{F}_6 , \mathcal{F}_7 , \mathcal{F}_8 , \mathcal{F}_9 , $\mathcal{$ **diage 1 improved classical MC would be use** S is the ferromagnetic Heisenberg model (33) at its critical (33) a **- improved classical MC would be useful** where D is a construction of D in D and D was used. Although D was used. Although D is a construction of D is a construction of D indicative, we cannot expect this estimate to be reliable, as it

ⁱ · *S*

S

&

the classical Heisenberg model (33) at its critical temperature. The

low value of **Pc** . The magnetic Bragg reflections are found at the magnetic Bragg reflect

Ω uantum and classical criticality in a **Quantum and classical criticality in a dimerized** au antum antiferromagnet Neutron-diffraction measurements at fields *H*#*Hc* revealed $\mathbf{f}^{\text{max}}_{\text{max}}$ **quantum antiferromagnet** $\overline{\mathbf{a}}$ riticality in a dimerize
|
|

P. Merchant¹, B. Normand², K. W. Krämer³, M. Boehm⁴, D. F. McMorrow¹ **P. Merchant¹, B. Normand², K. W. Krämer³, M. Boehm⁴, D. F. McMorrow¹ and Ch. Rüegg^{1,5,6*}**

tic neutron scattering !INS" measurements of the elementary

Energy (meV)

 $\frac{1}{\sqrt{2}}$

nature

imerized 3D Network of dimers contrast the field- and pressure-induced ordered phases of the system a couplings can be stated as \sim m_{R} μ_{R} and μ_{R} and μ_{R} and μ_{R} are all fields and μ_{R} and μ_{R} are all μ_{R} and μ_{R} are all μ_{R} and μ_{R} are all μ_{R} and μ_{R} are all

*et al.*¹⁸ have shown very recently by elastic neutronscattering measurements under a pressure of 1.48 GPa that

0.00

Universality of the Neel temperature in 3D dimerized systems? SONGBO JIN AND ANDERS W. SANDVIK PHYSICAL REVIEW B **85**, 020409(R) (2012)

1.0

ρ^s $\overline{}$

[S. Jin, AWS, PRB2012] $\begin{array}{ccc} \text{[0. 011, NWO, 11DZU1Z]} & \longrightarrow & \end{array}$

Determine the Neel ordering temperature **T_N** and the T=0 \uparrow ordered moment **m_s** for 3 different to the quantum-critical point $\frac{a}{a}$ dimerization and systems. Our results give a part of the system of t length *^L*, the number of spins is *^N* ⁼ *^L*³ in (a) and (b), and *^N* ⁼ ²*L*³

patterns

patterns **Example: Columnar dimers Example: Columnar compared with experimental can be contained with experimental 2.02.11 NetExample: Columnar dimers 85, 020409 (R) (2012)**

zation obtained in simulations with *T* = *J*1*/L* of the double-cube **EXECUTE MODEL AT A GIGAL AT A DIFFERENT COUPLINGS** VS PLESSURE HOL KILOV In this procedure of decoupling the classical and quantum **fluctuations, Couplings vs pressure not known experimentally** *Couplings vs pressure not known experimentally*

- much than the symbols. The synoid symbols is seen the fitting function used in the form of L \sim R \sim L \sim R \sim R extrapolations is *^a* ⁺ *b/L*² ⁺ *c/L*³ (where we exclude the linear term whuta hr - plot T_N vs ms to avoid this issue and study universality
- but how to normalize T_{N?} **Three normalizations**

 $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}})$ is defined by the subset of the subse

ms/S is *T* independent up to *TN* can be valid only if *TN* is iree normalizations which the energy scale in which the stations of the stations of the stations of the statio

- tash condition tash - weaker copling J₁
- sum J_s of couplings per spin
- as **graded to the density of the density o** - peak T^{*} of magnetic susceptibility

⟨**S**0⟩. The final magnetization curve is given by (*ms/S*)⟨**S**0⟩.

to the low-*T* weakly temperature-dependent form typical of

to explain this more quantitatively, by deriving the function \mathcal{E}

T* normalization is accessible experimentally *T*N(*p*) *TN /Js*

- some experimental susc. results available

0.10

Note that *ms* ! 1*/*2 for *S* = 1*/*2.

- neutron data analyzed with this normalization

*m*s(*p*)

2

,*T* = 1.8 K

each spin, i.e., *Js* = 5 + *g* for the columnar and staggered

here, a natural assumption is that *T* [∗] reflects an effective

Same features observed in models and experiment

- experimental slope about 25% lower if g-factor =2 assumed (what exactly is the g-factor?) (a) **b** indicate uncertainties in the resolution deconvolution. **c**, Complete experimental phase diagram, showing quantum disordered (QD), quantum critical FRITTE TO AND TO STOOM FOR 111 THO ROTO ATTACHLICATE ω magnetization for the three different dimerized models and ω *The property is the grading in the measured in units of the measured in units of the measured in units of the measured in the* antiferromagnets. The peak temperature *T* [∗], thus, reflects the short-distance energy scale at which antiferromagnetic corre-

More recent works to study log corrections, dynamics,.... Qin, Normand, Sandvik, Meng, PRB 2015, PRL 2017 an di bandar da band obtains in all cases for small to moderate *ms*, as indicate by fitted lines.

one high-*p* data point (open circle) taken from ref. 25 for an absolute calibration of *m*s. Data for *m*^s are normalized by *T*max =35 K, the maximum of the magnetic susceptibility13,16. Red lines in **d** and **e** represent scaling behaviour discussed in the text and error bars are the statistical uncertainties in the

Why the linear form $T_N = am_s$ (a=constant)?

The ordered state can be qualitatively described by mean field theory:

TN /Js

$$
H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j
$$

$$
H_0 = \left(\sum_i J_{i0}\right) \langle \vec{m}_s \rangle \cdot \vec{S}_0
$$

The order parameter m_s is reduced from its maximum value by two effects

- quantum fluctuations
- thermal fluctuations

Assume that these fluctuations decouple: **magnetization** for the three different dimensions and with the three distributions are all the three distributions are all three distributions are all three distributions of the th FIG. 4. (Color online) The Neel temperature ´ *TN* vs the sublattice

 $\langle m_s \rangle = m_0(T=0) f(T)$

In mean-field theory: $T_c \propto J_{\text{eff}} \rightarrow T_c \propto m_0$ Then we have an effective mean-field coupling $\, J_{\text{eff}} \propto m_{0} \,$ (c) the peak temperature *T* [∗] of the susceptibility. A linear dependence Note that *ms* ! 1*/*2 for *S* = 1*/*2.

The violations of the linear form indicate the temperature where quantum and thermal fluctuations cannot be decoupled

Logarithmic correction at the upper critical dimensionality square-root function [Eq. (23), green] and including the logarithmic **Logarithmic correction at the upper critical**

Mean-field theory is exact above the upper critical dimensionality du

b = trivial critical exponents \blacksquare trivial critical exportents

point to *gc* is *g* = 4*.*834. Lines show both the best fit by a pure

 \mathcal{P}_c distribution, a fit should be considered be considered be considered be considered be considered be considered by

- $-$ exactly at d=d_u there are logarithmic corrections to the power laws \blacksquare exactly at d=d_u there are logarithmic corrections to
- Test of expected log correction in the double-cube model $(d_u=4=3+1)$ Test of expected leg escreetion in the deuble **instruction of the COR region on the QC region on the QC regime on the QC region on the QC region on the COR** 16 12.1 J+I)

[Y. Q. Qin, B. Normand, A. W. Sandvik, Z.Y. Meng, PRB 2015]

 $\mathcal{F}_{\mathcal{A}}$, and the exponent of the expo

as far inside the Neel phase as ´ |*g* − *gc*|*/gc* ≈ 0*.*2, where the

More complex non-magnetic states; systems with 1 spin per unit cell

$$
H \ = \ J \sum_{\langle i,j \rangle} S_i \cdot S_j \ \ + \ \ g \times \cdots
$$

• **highly non-trivial non-magnetic ground states are possible, e.g.,**

- ➡ resonating valence-bond (RVB) spin liquid
- ➡ valence-bond solid (VBS)

Non-magnetic states often have natural descriptions with **valence bonds**

$$
\sum_i \sum_j = (\uparrow_i \downarrow_j - \downarrow_i \uparrow_j)/\sqrt{2}
$$

The basis including bonds of all lengths is **overcomplete** in the singlet sector

• non-magnetic states dominated by short bonds