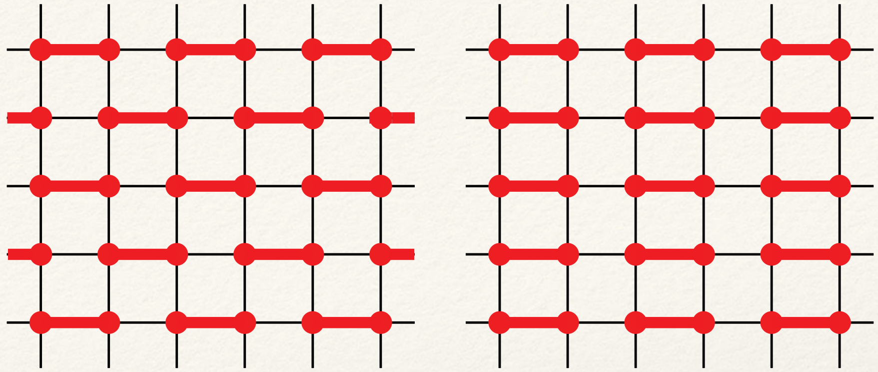
Case with more significant corrections

- common at quantum critical points



SDM CDM

S=1/2 Heisenberg model with
- columnar dimers (CDM)
- staggered dimers (SDM)

The SDM has been controversial
- O(3) or new universality class
- strange scaling behaviors

PHYSICAL REVIEW LETTERS **121**, 117202 (2018)

Anomalous Quantum-Critical Scaling Corrections in Two-Dimensional Antiferromagnets

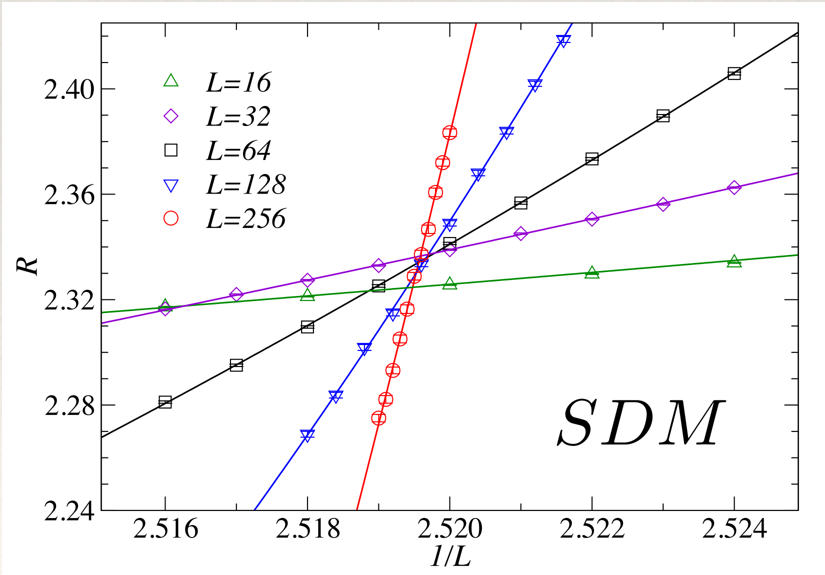
Nvsen Ma,[1,2,3] Phillip Weinberg,[3] Hui Shao,[4,3] Wenan Guo,[5,4] Dao-Xin Yao,[1,*] and Anders W. Sandvik[3,2,†]

Analyze critical behavior with two scaling corrections taken into account

$$O(g,L) = f[(g-g_c)L^{1/\nu}, \lambda_1 L^{-\omega_1}, \lambda_2 L^{-\omega_2}, \cdots]$$

Taylor expand, analyze crossing points for different dimensionless quantities

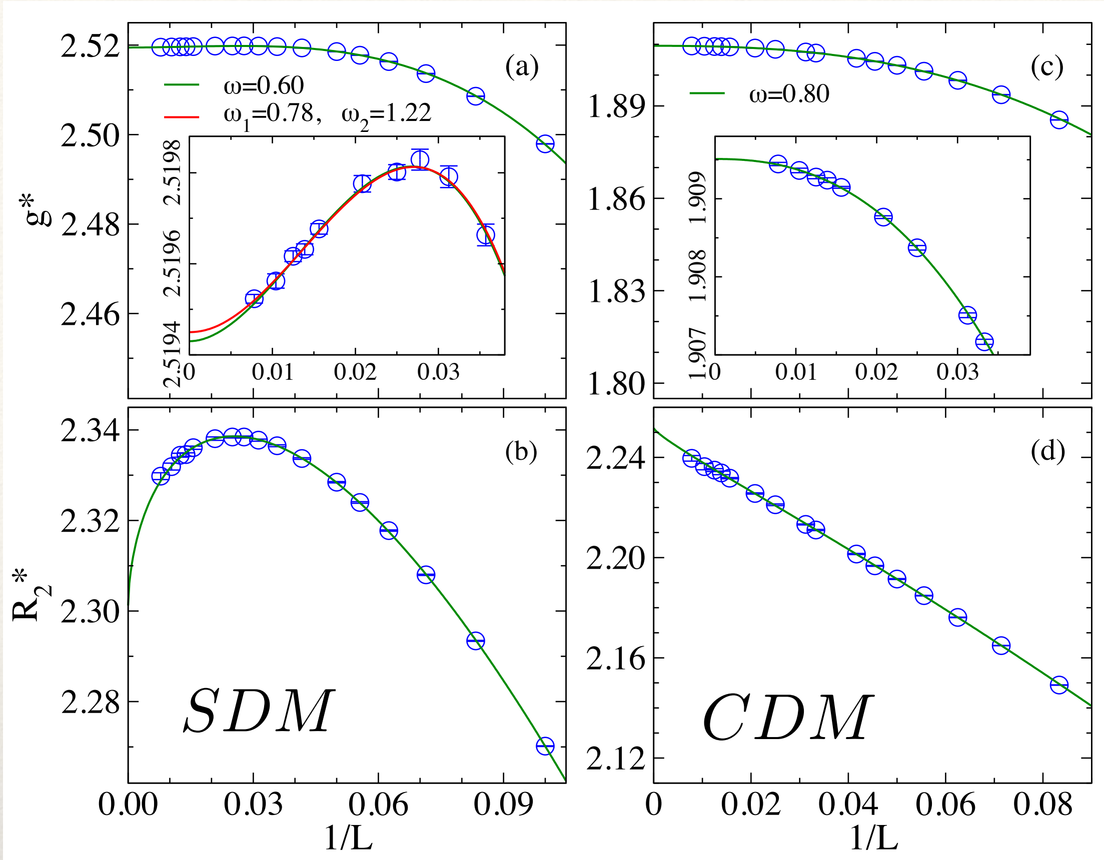
Compare CDM and SDM behaviors

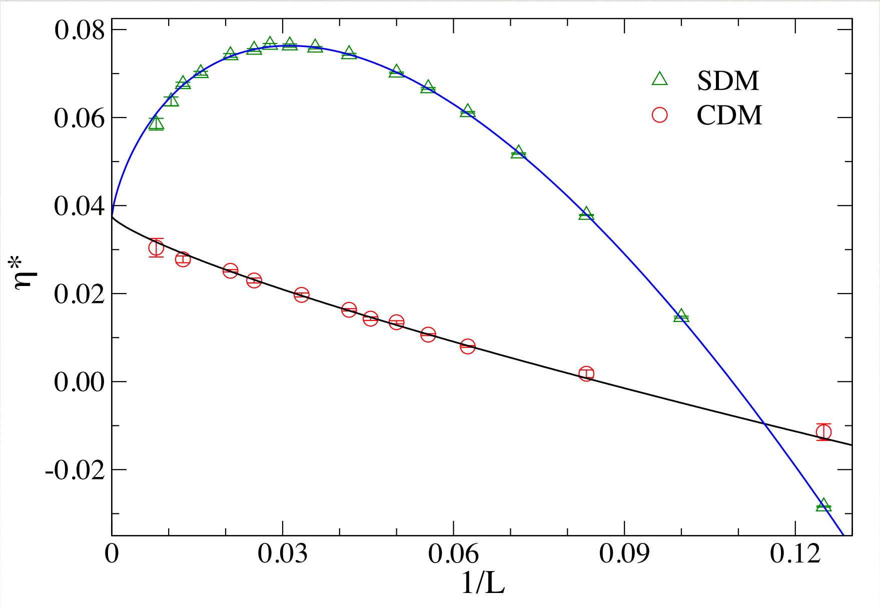


Leading-order cross-point shifts

$$g^*(L) = g_c + aL^{-\omega_1 - 1/\nu},$$
$$O^*(L) = O_c + bL^{-\omega_1},$$

- Works for CDM, $\omega_1 \approx 0.78$
- Two corrections needed for SDM $\omega_1 \approx 0.78$, $\omega_2 \approx 1.25$
- Fits within theory where the SDM field theory needs a new term (Fritz et al, PRB 2012)





Order parameter at the critical point

$$\langle m^2 \rangle_c \propto L^{-(1+\eta)}(1 + b_1 L^{-\omega_1} + b_2 L^{-\omega_2} + \ldots)$$

$$\eta^*(L) = \ln[\langle m^2(L)\rangle_c / \langle m^2(2L)\rangle_c]/\ln(2) - 1$$

$$\eta^*(L) = \eta + c_1 L^{-\omega_1} + c_2 L^{-\omega_2} + \ldots.$$

Field-theory aspects of the anomalous scaling [Fritz et al., PRB 2011]

The SDM leads to a cubic term in the field-theory action

$$\mathcal{S} = \frac{1}{2}\int d^D r\left[m_0\varphi_\alpha^2 + (\vec{\nabla}\varphi_\alpha)^2\right] + \frac{u_0}{4!}\int d^D r\left(\varphi_\alpha^2\right)^2 \; + i\gamma_0 \int d^D r\, \vec{\varphi}\cdot(\partial_x\vec{\varphi}\times\partial_y\vec{\varphi})$$

Is the cubic term relevant or irrelevant at the O(3) critical point?

$$C(\vec{r}) = \langle \mathcal{O}(\vec{r})\mathcal{O}(0)\rangle \propto \frac{1}{|\vec{r}|^{2\Delta_{\mathcal{O}}}} \qquad \mathcal{O}(\vec{r}) = \vec{\varphi}(\vec{r})\cdot(\partial_x\vec{\varphi}(\vec{r})\times\partial_y\vec{\varphi}(\vec{r}))$$

MC of the classical Heisenberg model to extract scaling dimension

$$H = -J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j$$

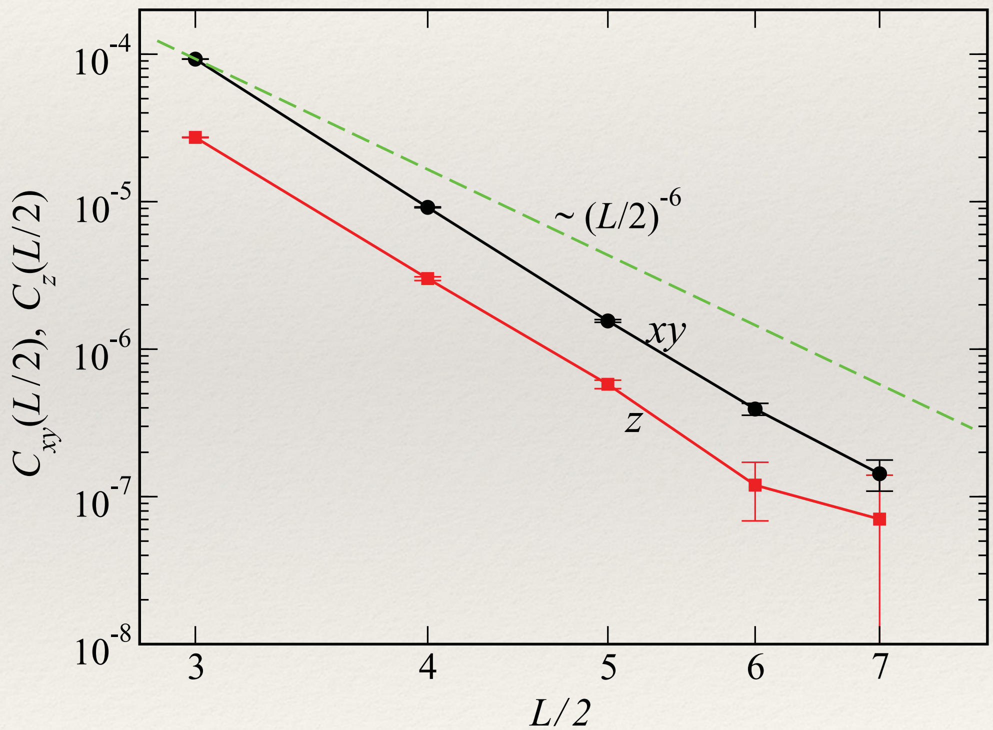
$$\mathcal{O}_{i,\text{lattice}} = \vec{S}_i\cdot\left(\vec{S}_{i+e_x}\times\vec{S}_{i+e_y}\right)$$

They found Δ_o ~ 3.2 … 3.5

- corresponds to ω ~ 0.2 - 0.5
- would be leading correction

Inconsistent with SDM QMC

- ω ~ 1.3
- second correction



Conformal bootstrap method: There is an operator with ω ~ 1.4

- improved classical MC would be useful

nature physics
ARTICLES
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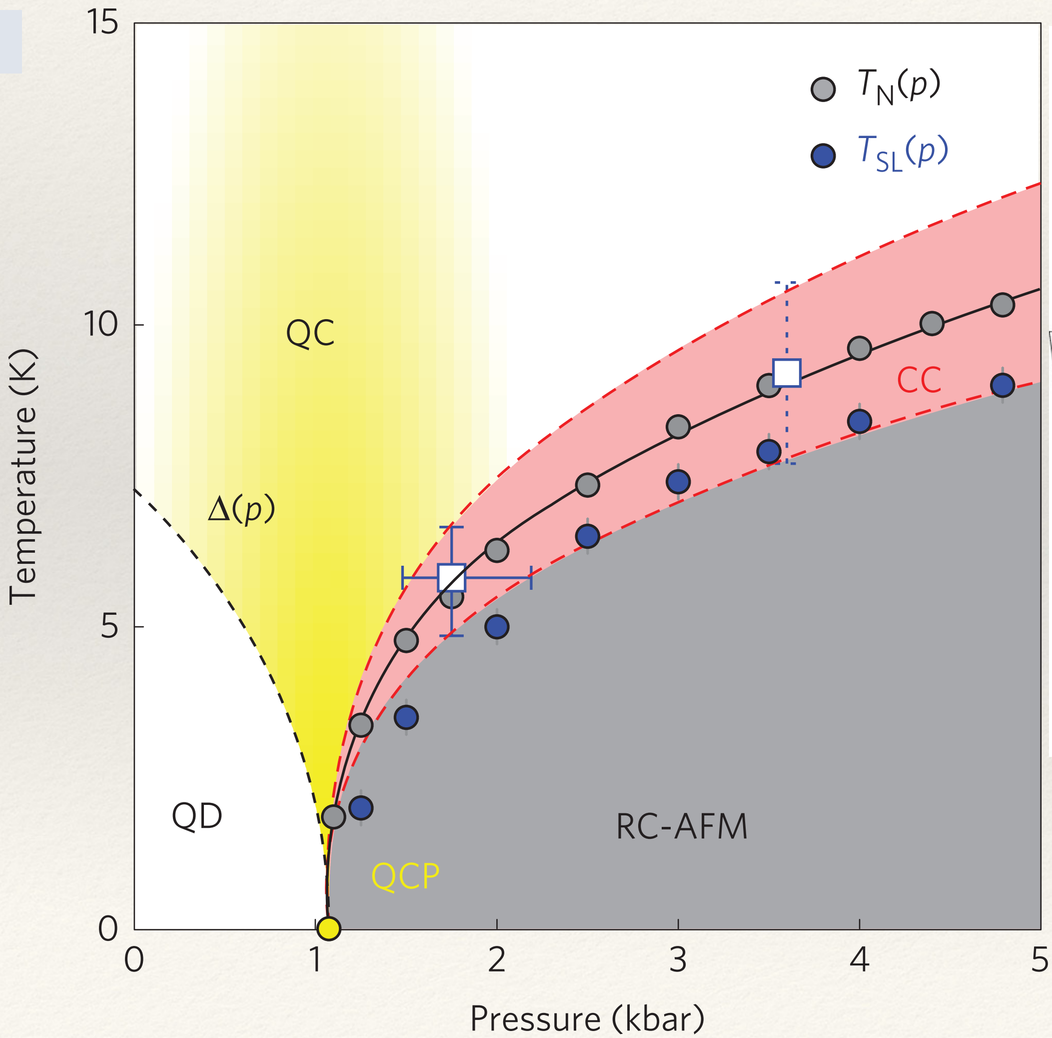
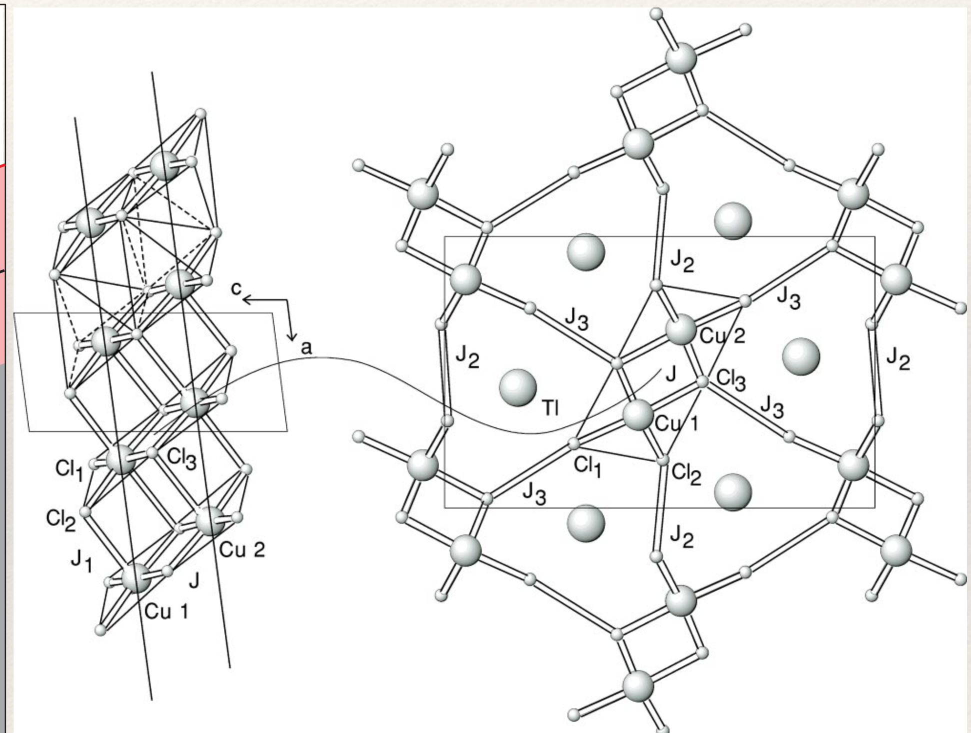
$TlCuCl_3$

Quantum and classical criticality in a dimerized quantum antiferromagnet

P. Merchant[1], B. Normand[2], K. W. Krämer[3], M. Boehm[4], D. F. McMorrow[1] and Ch. Rüegg[1,5,6★]

3D Network of dimers
- couplings can be changed by pressure



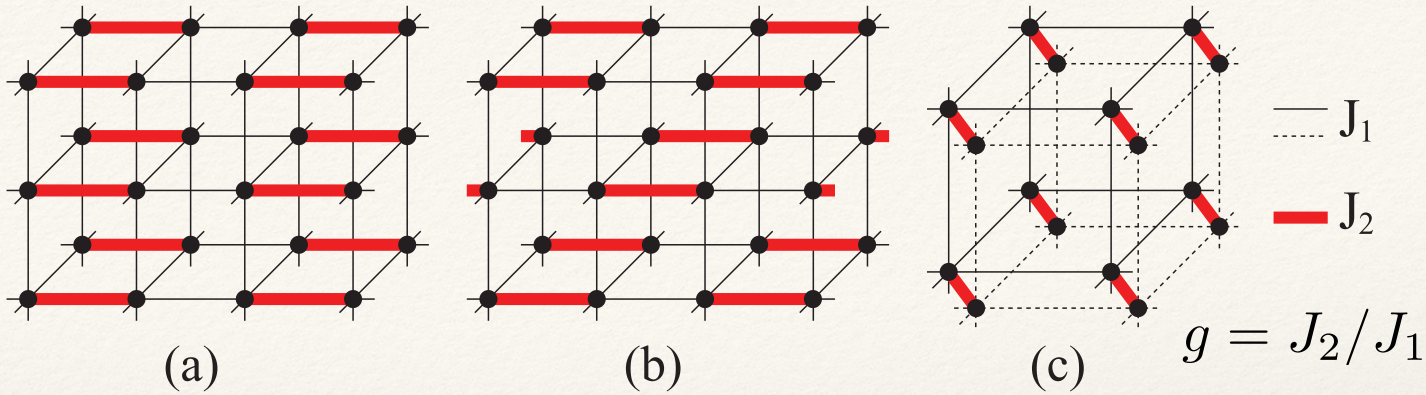


From: M Matsumoto, B Normand, TM Rice, M Sigrist, PRB (2004)

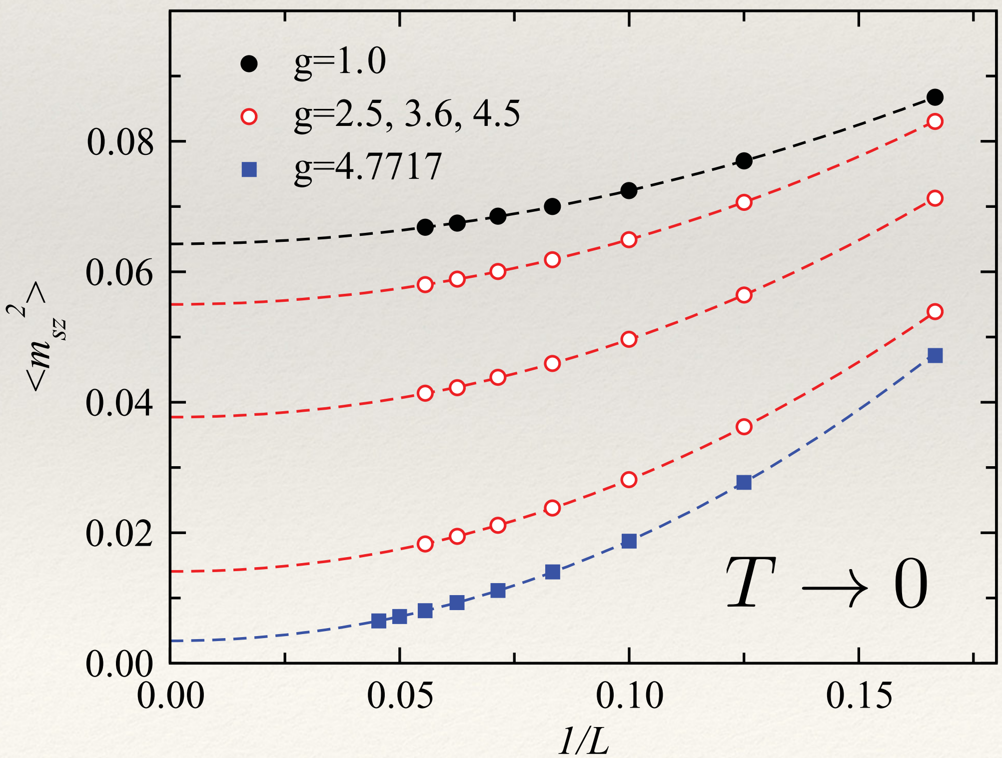
Universality of the Neel temperature in 3D dimerized systems?

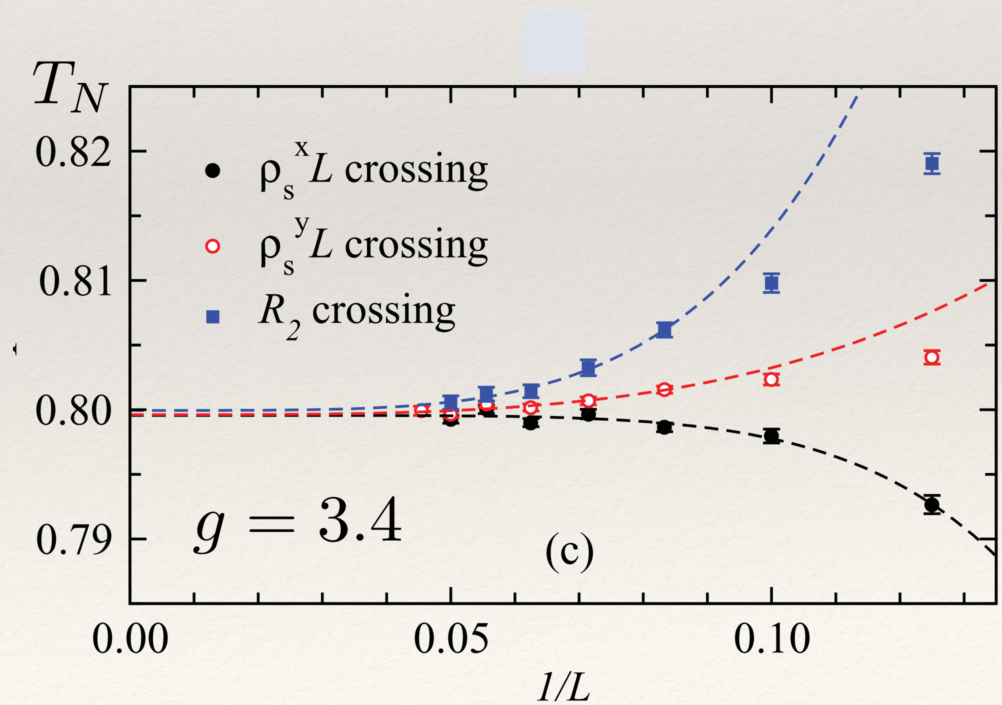
[S. Jin, AWS, PRB2012]

Determine the Neel ordering temperature T_N and the T=0 ordered moment m_s for 3 different dimerization patterns



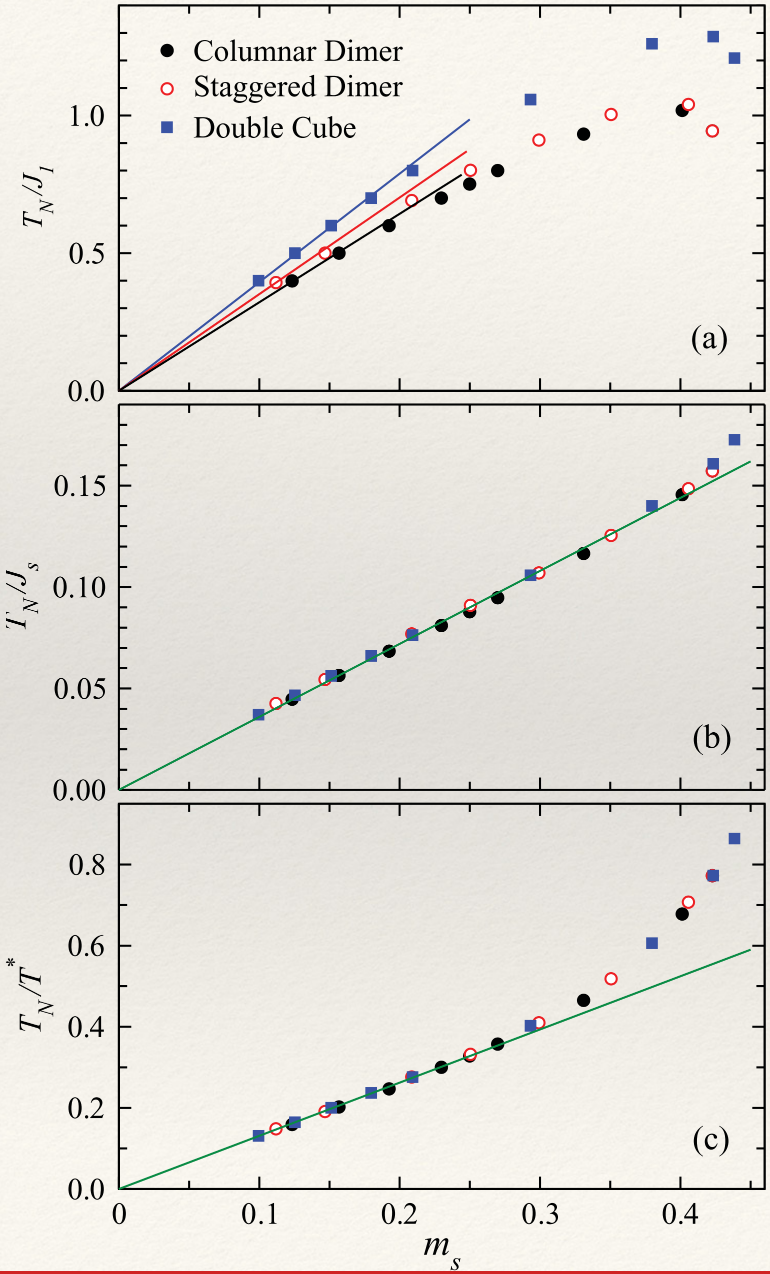
Example: Columnar dimers





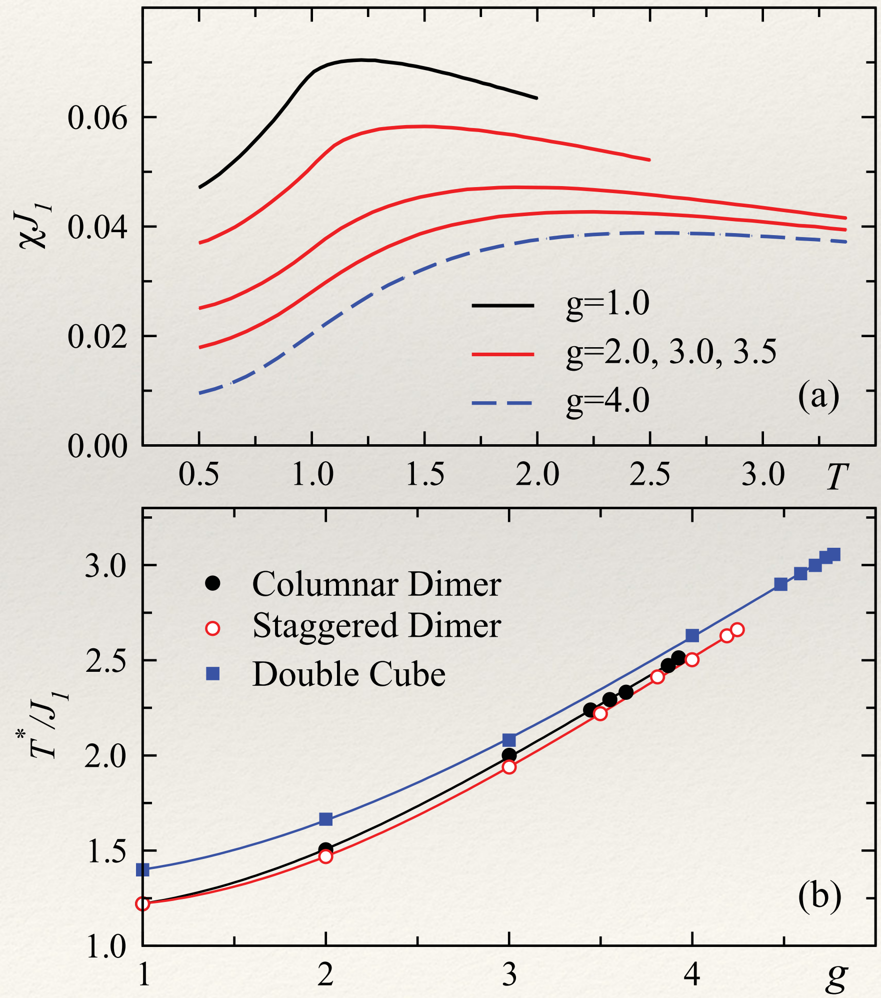
Couplings vs pressure not known experimentally

- plot T_N vs m_s to avoid this issue and study universality
- but how to normalize T_N?



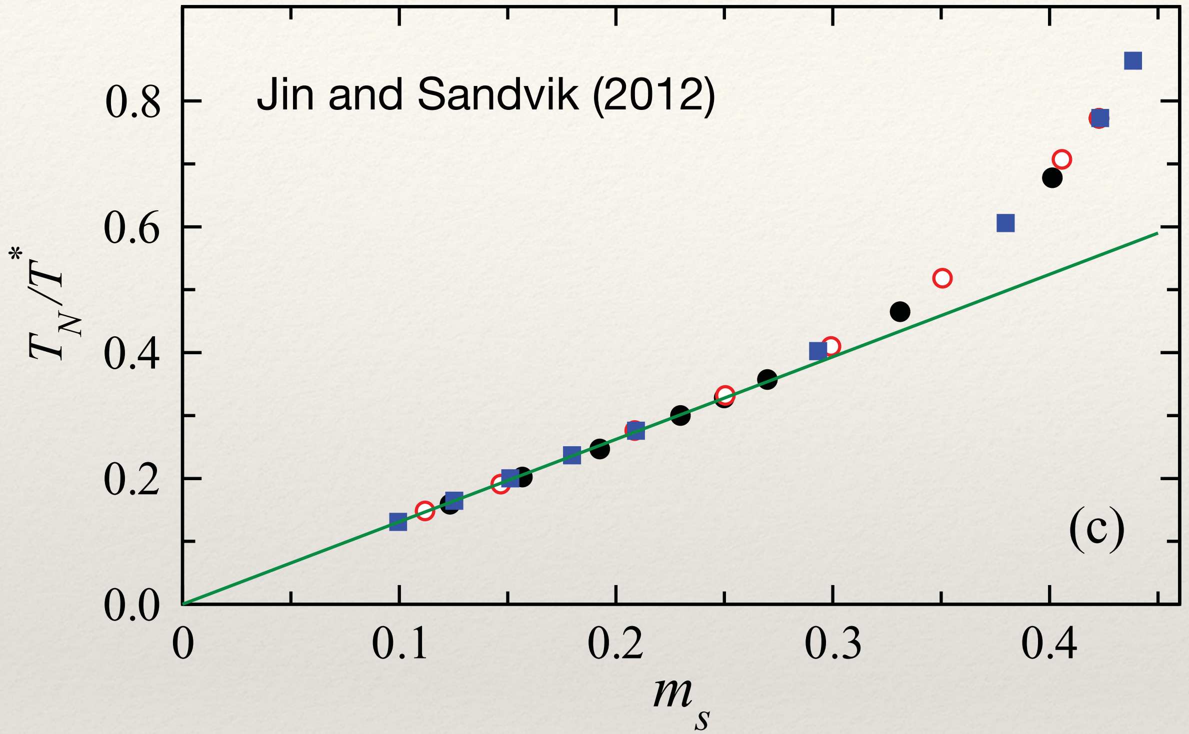
Three normalizations

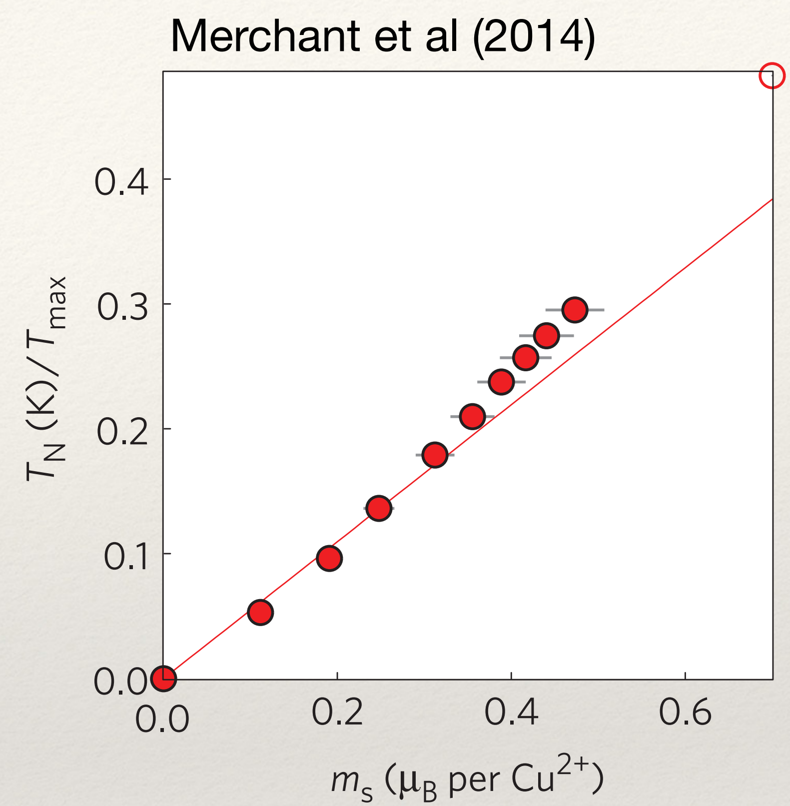
- weaker copling J_1
- sum J_s of couplings per spin
- peak T^* of magnetic susceptibility



T* normalization is accessible experimentally

- some experimental susc. results available
- neutron data analyzed with this normalization





Same features observed in models and experiment

- experimental slope about 25% lower if g-factor =2 assumed
  (what exactly is the g-factor?)

More recent works to study log corrections, dynamics,….

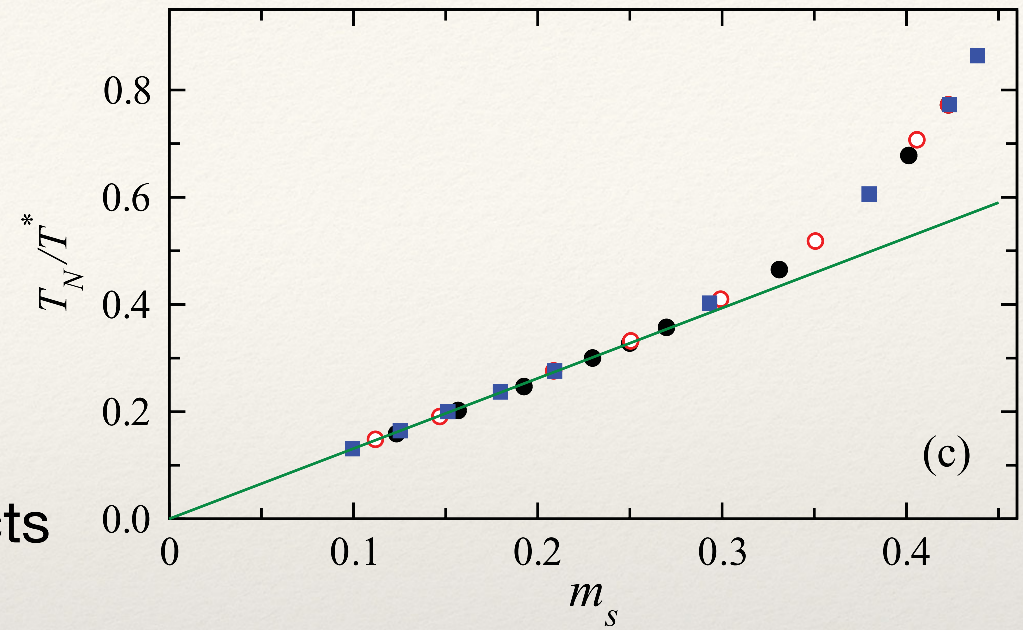
Qin, Normand, Sandvik, Meng, PRB 2015, PRL 2017

Why the linear form $T_N = am_s$ (a=constant)?

The ordered state can be qualitatively described by mean field theory:

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$H_0 = \left(\sum_i J_{i0} \right) \langle \vec{m}_s \rangle \cdot \vec{S}_0$$



The order parameter m_s is reduced from its maximum value by two effects

- quantum fluctuations
- thermal fluctuations

Assume that these fluctuations decouple:

$$\langle m_s \rangle = m_0(T=0) f(T)$$

Then we have an effective mean-field coupling $J_{\text{eff}} \propto m_0$

In mean-field theory: $T_c \propto J_{\text{eff}} \quad \rightarrow \quad T_c \propto m_0$

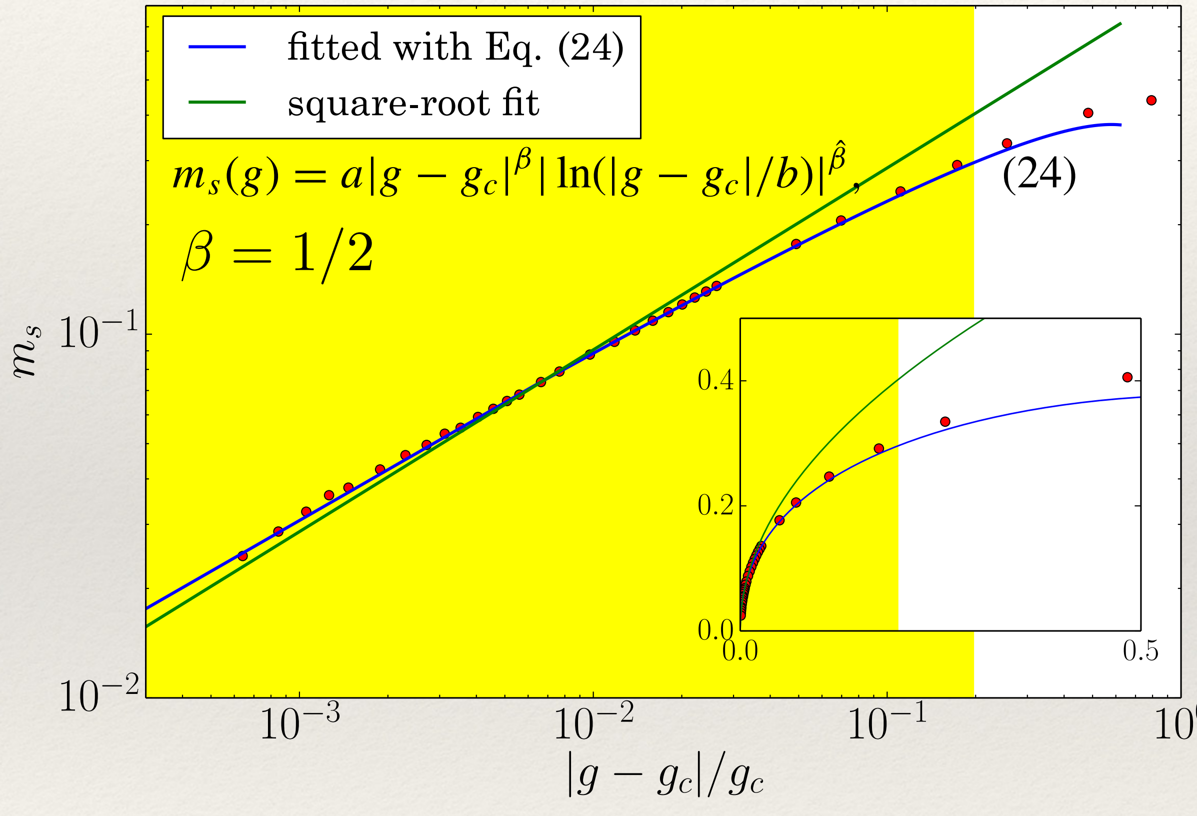
The violations of the linear form indicate the temperature where quantum and thermal fluctuations cannot be decoupled

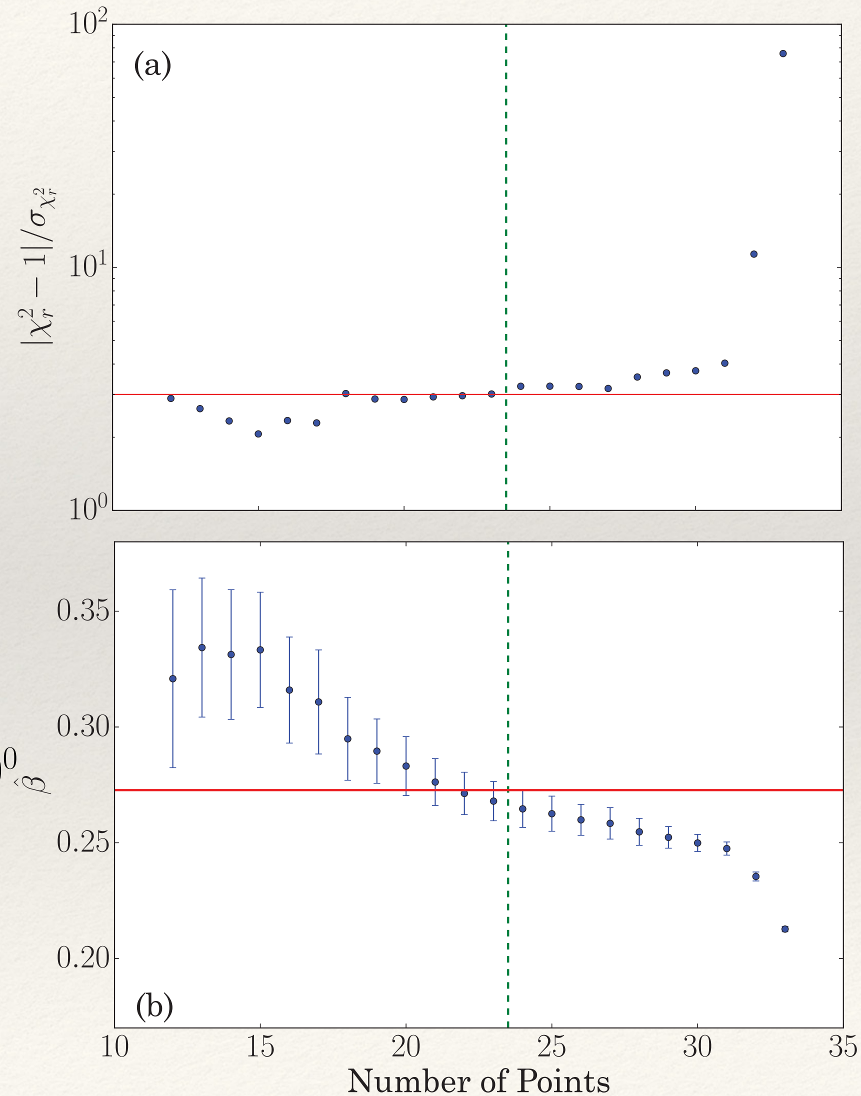
Logarithmic correction at the upper critical dimensionality

Mean-field theory is exact above the upper critical dimensionality d_u

- trivial critical exponents
- exactly at $d=d_u$ there are logarithmic corrections to the power laws

Test of expected log correction in the double-cube model ($d_u=4=3+1$)
[Y. Q. Qin, B. Normand, A. W. Sandvik, Z.Y. Meng, PRB 2015]





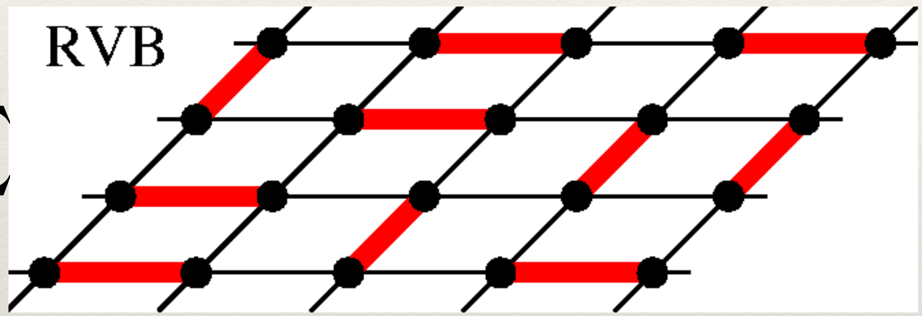
Conclusion: Log corrections can be detected if the leading exponent is known/fixed

More complex non-magnetic states; systems with 1 spin per unit cell

$$\mathbf{H} = \mathbf{J}\sum_{\langle \mathbf{i},\mathbf{j}\rangle} \mathbf{S_i}\cdot\mathbf{S_j} + \mathbf{g}\times\cdots$$

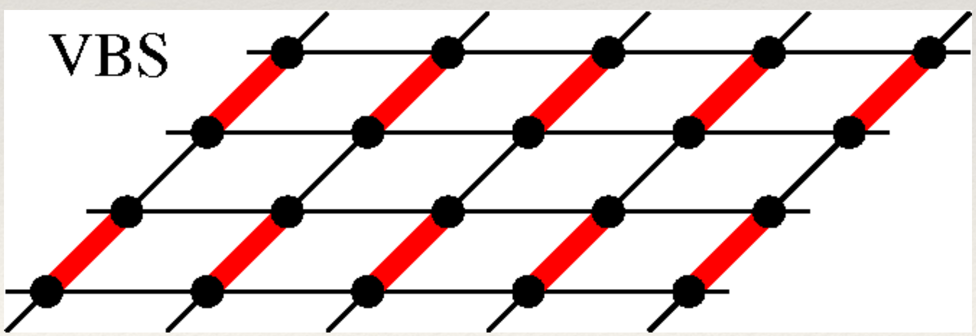
- **highly non-trivial non-magnetic ground states are possible, e.g.,**
  - ➡ resonating valence-bond (RVB) spin liquid
  - ➡ valence-bond solid (VBS)

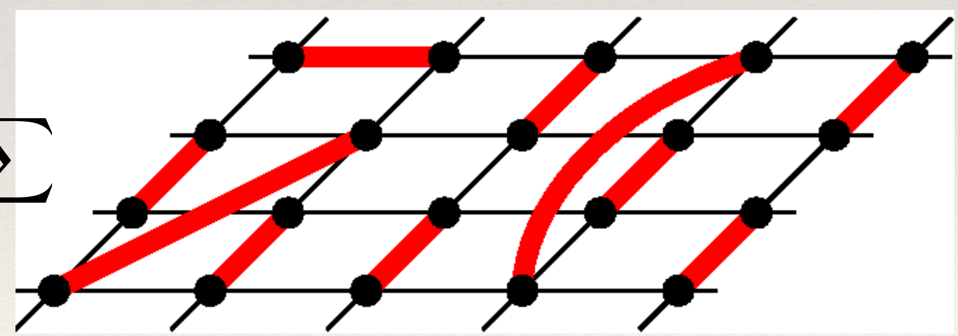
Non-magnetic states often have natural descriptions with **valence bonds**

$\sum$ 

$= (\uparrow_i\downarrow_j - \downarrow_i\uparrow_j)/\sqrt{2}$

The basis including bonds of all lengths is **overcomplete** in the singlet sector



$\sum$ 

- non-magnetic states dominated by short bonds